ALGEBRA

1 Evaluate

a 8^2 **b** 6^3 **c** 7^0 **d** $(-5)^4$ **e** $(-3)^5$ **f** $(\frac{1}{2})^4$

g $(\frac{2}{3})^3$ **h** $(-\frac{1}{4})^3$ **i** $(1\frac{1}{3})^2$ **j** $(1\frac{1}{2})^4$ **k** $(0.1)^5$ **l** $(-0.2)^3$

2 Write in the form 2^n

a $2^5 \times 2^3$ **b** 2×2^6 **c** 1 **d** $2^6 \div 2^2$ **e** $2^{15} \div 2^6$ **f** $(2^7)^2$

3 Simplify

a $2p^2 \times 4p^5$ **b** $x^2 \times x^3 \times x^5$ **c** $12n^7 \div 2n^2$ **d** $(y^3)^4$

e $(2b)^3 \div 4b^2$ **f** $p^3q \times pq^2$ **g** $x^4y^3 \div xy^2$ **h** $2r^2s \times 3s^2$

i $6x^5y^8 \div 3x^2y$ **j** $6a^4b^5 \times \frac{2}{3}ab^3$ **k** $(5rs^2)^3 \div (10rs)^2$ **l** $3p^4q^3 \div \frac{1}{5}pq^2$

4 Evaluate

a 3^{-2} **b** $(\frac{2}{5})^0$ **c** $(-2)^{-6}$ **d** $(\frac{1}{6})^{-2}$ **e** $(1\frac{1}{2})^{-3}$ **f** $9^{\frac{1}{2}}$

g $16^{\frac{1}{4}}$ **h** $(-27)^{\frac{1}{3}}$ **i** $(\frac{1}{49})^{\frac{1}{2}}$ **j** $125^{\frac{1}{3}}$ **k** $(\frac{4}{9})^{\frac{1}{2}}$ **l** $36^{-\frac{1}{2}}$

m $81^{-\frac{1}{4}}$ **n** $(-64)^{-\frac{1}{3}}$ **o** $(\frac{1}{32})^{-\frac{1}{5}}$ **p** $(-\frac{8}{125})^{\frac{1}{3}}$ **q** $(2\frac{1}{4})^{\frac{1}{2}}$ **r** $(3\frac{3}{8})^{-\frac{1}{3}}$

5 Evaluate

a $4^{\frac{3}{2}}$ **b** $27^{\frac{2}{3}}$ **c** $16^{\frac{3}{4}}$ **d** $(-125)^{\frac{2}{3}}$ **e** $9^{\frac{5}{2}}$ **f** $8^{-\frac{2}{3}}$

g $36^{-\frac{3}{2}}$ **h** $(\frac{1}{8})^{\frac{4}{3}}$ **i** $(\frac{4}{9})^{\frac{3}{2}}$ **j** $(\frac{1}{216})^{-\frac{2}{3}}$ **k** $(\frac{9}{16})^{-\frac{3}{2}}$ **l** $(-\frac{27}{64})^{\frac{4}{3}}$

m $(0.04)^{\frac{1}{2}}$ **n** $(2.25)^{-\frac{3}{2}}$ **o** $(0.064)^{\frac{2}{3}}$ **p** $(1\frac{9}{16})^{-\frac{3}{2}}$ **q** $(5\frac{1}{16})^{\frac{3}{4}}$ **r** $(2\frac{10}{27})^{-\frac{4}{3}}$

6 Work out

a $4^{\frac{1}{2}} \times 27^{\frac{1}{3}}$ **b** $16^{\frac{1}{4}} + 25^{\frac{1}{2}}$ **c** $8^{-\frac{1}{3}} \div 36^{\frac{1}{2}}$ **d** $(-64)^{\frac{1}{3}} \times 9^{\frac{3}{2}}$

e $(\frac{1}{3})^{-2} - (-8)^{\frac{1}{3}}$ **f** $(\frac{1}{25})^{\frac{1}{2}} \times (\frac{1}{4})^{-2}$ **g** $81^{\frac{3}{4}} - (\frac{1}{49})^{-\frac{1}{2}}$ **h** $(\frac{1}{27})^{-\frac{1}{3}} \times (\frac{4}{9})^{-\frac{3}{2}}$

i $(\frac{1}{9})^{-\frac{1}{2}} \times (-32)^{\frac{3}{5}}$ **j** $(121)^{0.5} + (32)^{0.2}$ **k** $(100)^{0.5} \div (0.25)^{1.5}$ **l** $(16)^{-0.25} \times (243)^{0.4}$

7 Simplify

a $x^8 \times x^{-6}$ **b** $y^{-2} \times y^{-4}$ **c** $6p^3 \div 2p^7$ **d** $(2x^{-4})^3$

e $y^3 \times y^{-\frac{1}{2}}$ **f** $2b^{\frac{2}{3}} \times 4b^{\frac{1}{4}}$ **g** $x^{\frac{3}{5}} \div x^{\frac{1}{3}}$ **h** $a^{\frac{1}{2}} \div a^{\frac{4}{3}}$

i $p^{\frac{1}{4}} \div p^{-\frac{1}{5}}$ **j** $(3x^{\frac{2}{5}})^2$ **k** $y \times y^{\frac{5}{6}} \times y^{-\frac{3}{2}}$ **l** $4t^{\frac{3}{2}} \div 12t^{\frac{1}{2}}$

m $\dfrac{b^2 \times b^{\frac{1}{4}}}{b^{\frac{1}{2}}}$ **n** $\dfrac{y^{\frac{1}{2}} \times y^{\frac{1}{3}}}{y}$ **o** $\dfrac{4x^{\frac{2}{3}} \times 3x^{-\frac{1}{6}}}{6x^{\frac{3}{4}}}$ **p** $\dfrac{2a \times a^{\frac{3}{4}}}{8a^{-\frac{1}{2}}}$

8 Solve each equation.

a $x^{\frac{1}{2}} = 6$ **b** $x^{\frac{1}{3}} = 5$ **c** $x^{-\frac{1}{2}} = 2$ **d** $x^{-\frac{1}{4}} = \frac{1}{3}$

e $x^{\frac{3}{2}} = 8$ **f** $x^{\frac{2}{3}} = 16$ **g** $x^{\frac{4}{3}} = 81$ **h** $x^{-\frac{3}{2}} = 27$

9 Express in the form x^k

a $\sqrt{x}$ **b** $\frac{1}{\sqrt[3]{x}}$ **c** $x^2 \times \sqrt{x}$ **d** $\frac{\sqrt[4]{x}}{x}$

e $\sqrt{x^3}$ **f** $\sqrt{x} \times \sqrt[3]{x}$ **g** $(\sqrt{x})^5$ **h** $\sqrt[3]{x^2} \times (\sqrt{x})^3$

10 Express each of the following in the form ax^b, where a and b are rational constants.

a $\frac{4}{\sqrt{x}}$ **b** $\frac{1}{2x}$ **c** $\frac{3}{4x^3}$ **d** $\frac{1}{(3x)^2}$ **e** $\frac{2}{5\sqrt[3]{x}}$ **f** $\frac{1}{\sqrt{9x^3}}$

11 Express in the form 2^k

a 8^2 **b** $(\frac{1}{4})^{-2}$ **c** $(\frac{1}{2})^{\frac{1}{3}}$ **d** $16^{-\frac{1}{6}}$ **e** $8^{\frac{2}{5}}$ **f** $(\frac{1}{32})^{-3}$

12 Express each of the following in the form 3^y, where y is a function of x.

a 9^x **b** 81^{x+1} **c** $27^{\frac{x}{4}}$ **d** $(\frac{1}{3})^x$ **e** 9^{2x-1} **f** $(\frac{1}{27})^{x+2}$

13 Given that $y = 2^x$, express each of the following in terms of y.

a 2^{x+1} **b** 2^{x-2} **c** 2^{2x} **d** 8^x **e** 2^{4x+3} **f** $(\frac{1}{2})^{x-3}$

14 Find the value of x such that

a $2^x = 64$ **b** $5^{x-1} = 125$ **c** $3^{x+4} - 27 = 0$ **d** $8^x - 2 = 0$

e $3^{2x-1} = 9$ **f** $16 - 4^{3x-2} = 0$ **g** $9^{x-2} = 27$ **h** $8^{2x+1} = 16$

i $49^{x+1} = \sqrt{7}$ **j** $3^{3x-2} = \sqrt[3]{9}$ **k** $(\frac{1}{6})^{x+3} = 36$ **l** $(\frac{1}{2})^{3x-1} = 8$

15 Solve each equation.

a $2^{x+3} = 4^x$ **b** $5^{3x} = 25^{x+1}$ **c** $9^{2x} = 3^{x-3}$ **d** $16^x = 4^{1-x}$

e $4^{x+2} = 8^x$ **f** $27^{2x} = 9^{3-x}$ **g** $6^{3x-1} = 36^{x+2}$ **h** $8^x = 16^{2x-1}$

i $125^x = 5^{x-3}$ **j** $(\frac{1}{3})^x = 3^{x-4}$ **k** $(\frac{1}{2})^{1-x} = (\frac{1}{8})^{2x}$ **l** $(\frac{1}{4})^{x+1} = 8^x$

16 Expand and simplify

a $x(x^2 - x^{-1})$ **b** $2x^3(x^{-1} + 3)$ **c** $x^{-1}(3x - x^3)$ **d** $4x^{-2}(3x^5 + 2x^3)$

e $\frac{1}{2}x^2(6x + 4x^{-1})$ **f** $3x^{\frac{1}{2}}(x^{-\frac{1}{2}} - x^{\frac{3}{2}})$ **g** $x^{-\frac{3}{2}}(5x^2 + x^{\frac{7}{2}})$ **h** $x^{\frac{1}{3}}(3x^{\frac{5}{3}} - x^{-\frac{4}{3}})$

i $(x^2 + 1)(x^4 - 3)$ **j** $(2x^5 + x)(x^4 + 3)$ **k** $(x^2 - 2x^{-1})(x - x^{-2})$ **l** $(x^2 - x^{\frac{3}{2}})(x - x^{\frac{1}{2}})$

17 Simplify

a $\frac{x^3 + 2x}{x}$ **b** $\frac{4t^5 - 6t^3}{2t^2}$ **c** $\frac{x^{\frac{3}{2}} - 3x}{x^{\frac{1}{2}}}$ **d** $\frac{y^2(y^3 - 6)}{3y}$

e $\frac{p + p^{\frac{3}{2}}}{p^{\frac{3}{4}}}$ **f** $\frac{8w - 2w^{\frac{1}{2}}}{4w^{-\frac{1}{2}}}$ **g** $\frac{x + 1}{x^{\frac{1}{2}} + x^{-\frac{1}{2}}}$ **h** $\frac{2t^3 - 4t}{t^{\frac{3}{2}} - 2t^{-\frac{1}{2}}}$

ALGEBRA

1 Evaluate

a $\sqrt{49}$ **b** $\sqrt{121}$ **c** $\sqrt{\frac{1}{9}}$ **d** $\sqrt{\frac{4}{25}}$ **e** $\sqrt{0.01}$ **f** $\sqrt{0.09}$

g $\sqrt[3]{8}$ **h** $\sqrt[3]{1000}$ **i** $\sqrt[4]{81}$ **j** $\sqrt{1\frac{9}{16}}$ **k** $\sqrt[3]{0.125}$ **l** $\sqrt[3]{15\frac{5}{8}}$

2 Simplify

a $\sqrt{7} \times \sqrt{7}$ **b** $4\sqrt{5} \times \sqrt{5}$ **c** $(3\sqrt{3})^2$ **d** $(\sqrt{6})^4$

e $(\sqrt{2})^5$ **f** $(2\sqrt{3})^3$ **g** $\sqrt{2} \times \sqrt{8}$ **h** $2\sqrt{3} \times \sqrt{27}$

i $\dfrac{\sqrt{32}}{\sqrt{2}}$ **j** $\dfrac{\sqrt{3}}{\sqrt{12}}$ **k** $(\sqrt[3]{6})^3$ **l** $(3\sqrt[3]{2})^3$

3 Express in the form $k\sqrt{2}$

a $\sqrt{18}$ **b** $\sqrt{50}$ **c** $\sqrt{8}$ **d** $\sqrt{98}$ **e** $\sqrt{200}$ **f** $\sqrt{162}$

4 Simplify

a $\sqrt{12}$ **b** $\sqrt{28}$ **c** $\sqrt{80}$ **d** $\sqrt{27}$ **e** $\sqrt{24}$ **f** $\sqrt{128}$

g $\sqrt{45}$ **h** $\sqrt{40}$ **i** $\sqrt{75}$ **j** $\sqrt{112}$ **k** $\sqrt{99}$ **l** $\sqrt{147}$

m $\sqrt{216}$ **n** $\sqrt{800}$ **o** $\sqrt{180}$ **p** $\sqrt{60}$ **q** $\sqrt{363}$ **r** $\sqrt{208}$

5 Simplify

a $\sqrt{18} + \sqrt{50}$ **b** $\sqrt{48} - \sqrt{27}$ **c** $2\sqrt{8} + \sqrt{72}$

d $\sqrt{360} - 2\sqrt{40}$ **e** $2\sqrt{5} - \sqrt{45} + 3\sqrt{20}$ **f** $\sqrt{24} + \sqrt{150} - 2\sqrt{96}$

6 Express in the form $a + b\sqrt{3}$

a $\sqrt{3}(2 + \sqrt{3})$ **b** $4 - \sqrt{3} - 2(1 - \sqrt{3})$ **c** $(1 + \sqrt{3})(2 + \sqrt{3})$

d $(4 + \sqrt{3})(1 + 2\sqrt{3})$ **e** $(3\sqrt{3} - 4)^2$ **f** $(3\sqrt{3} + 1)(2 - 5\sqrt{3})$

7 Simplify

a $(\sqrt{5} + 1)(2\sqrt{5} + 3)$ **b** $(1 - \sqrt{2})(4\sqrt{2} - 3)$ **c** $(2\sqrt{7} + 3)^2$

d $(3\sqrt{2} - 1)(2\sqrt{2} + 5)$ **e** $(\sqrt{5} - \sqrt{2})(\sqrt{5} + 2\sqrt{2})$ **f** $(3 - \sqrt{8})(4 + \sqrt{2})$

8 Express each of the following as simply as possible with a rational denominator.

a $\dfrac{1}{\sqrt{5}}$ **b** $\dfrac{2}{\sqrt{3}}$ **c** $\dfrac{1}{\sqrt{8}}$ **d** $\dfrac{14}{\sqrt{7}}$ **e** $\dfrac{3\sqrt{2}}{\sqrt{3}}$ **f** $\dfrac{\sqrt{5}}{\sqrt{15}}$

g $\dfrac{1}{3\sqrt{7}}$ **h** $\dfrac{12}{\sqrt{72}}$ **i** $\dfrac{1}{\sqrt{80}}$ **j** $\dfrac{3}{2\sqrt{54}}$ **k** $\dfrac{4\sqrt{20}}{3\sqrt{18}}$ **l** $\dfrac{3\sqrt{175}}{2\sqrt{27}}$

9 Simplify

a $\sqrt{8} + \dfrac{6}{\sqrt{2}}$ **b** $\sqrt{48} - \dfrac{10}{\sqrt{3}}$ **c** $\dfrac{6-\sqrt{8}}{\sqrt{2}}$

d $\dfrac{\sqrt{45}-5}{\sqrt{20}}$ **e** $\dfrac{1}{\sqrt{18}} + \dfrac{1}{\sqrt{32}}$ **f** $\dfrac{2}{\sqrt{3}} - \dfrac{\sqrt{6}}{\sqrt{72}}$

10 Solve each equation, giving your answers as simply as possible in terms of surds.

a $x(x + 4) = 4(x + 8)$ **b** $x - \sqrt{48} = 2\sqrt{3} - 2x$

c $x\sqrt{18} - 4 = \sqrt{8}$ **d** $x\sqrt{5} + 2 = \sqrt{20}\,(x - 1)$

11 **a** Simplify $(2 - \sqrt{3})(2 + \sqrt{3})$.

b Express $\dfrac{2}{2-\sqrt{3}}$ in the form $a + b\sqrt{3}$.

12 Express each of the following as simply as possible with a rational denominator.

a $\dfrac{1}{\sqrt{2}+1}$ **b** $\dfrac{4}{\sqrt{3}-1}$ **c** $\dfrac{1}{\sqrt{6}-2}$ **d** $\dfrac{3}{2+\sqrt{3}}$

e $\dfrac{1}{2+\sqrt{5}}$ **f** $\dfrac{\sqrt{2}}{\sqrt{2}-1}$ **g** $\dfrac{6}{\sqrt{7}+3}$ **h** $\dfrac{1}{3+2\sqrt{2}}$

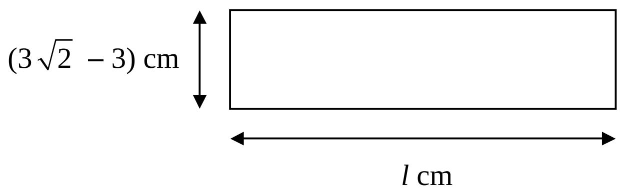
i $\dfrac{1}{4-2\sqrt{3}}$ **j** $\dfrac{3}{3\sqrt{2}+4}$ **k** $\dfrac{2\sqrt{3}}{7-4\sqrt{3}}$ **l** $\dfrac{6}{\sqrt{5}-\sqrt{3}}$

13 Solve the equation

$$3x = \sqrt{5}\,(x + 2),$$

giving your answer in the form $a + b\sqrt{5}$, where a and b are rational.

14



The diagram shows a rectangle measuring $(3\sqrt{2} - 3)$ cm by l cm.
Given that the area of the rectangle is 6 cm^2, find the exact value of l in its simplest form.

15 Express each of the following as simply as possible with a rational denominator.

a $\dfrac{\sqrt{2}}{\sqrt{2}+\sqrt{6}}$ **b** $\dfrac{1+\sqrt{3}}{2+\sqrt{3}}$ **c** $\dfrac{1+\sqrt{10}}{\sqrt{10}-3}$ **d** $\dfrac{3-\sqrt{2}}{4+3\sqrt{2}}$

e $\dfrac{1-\sqrt{2}}{3-\sqrt{8}}$ **f** $\dfrac{\sqrt{3}-5}{2\sqrt{3}-4}$ **g** $\dfrac{\sqrt{12}+3}{3-\sqrt{3}}$ **h** $\dfrac{3\sqrt{7}-2}{2\sqrt{7}-5}$

ALGEBRA

1 Express each of the following in the form $a\sqrt{2} + b\sqrt{3}$, where a and b are integers.

a $\sqrt{27} + 2\sqrt{50}$

b $\sqrt{6}(\sqrt{3} - \sqrt{8})$

2 Given that $x > 0$, find in the form $k\sqrt{3}$ the value of x such that

$$x(x - 2) = 2(6 - x).$$

3 Solve the equation

$$25^x = 5^{4x + 1}.$$

4 **a** Express $\sqrt[3]{24}$ in the form $k\sqrt[3]{3}$.

b Find the integer n such that

$$\sqrt[3]{24} + \sqrt[3]{81} = \sqrt[3]{n}.$$

5 Show that

$$\frac{10\sqrt{3}}{\sqrt{15}} + \frac{4}{\sqrt{5} - \sqrt{7}}$$

can be written in the form $k\sqrt{7}$, where k is an integer to be found.

6 Showing your method clearly,

a express $\sqrt{37.5}$ in the form $a\sqrt{6}$,

b express $\sqrt{9\tfrac{3}{5}} - \sqrt{6\tfrac{2}{3}}$ in the form $b\sqrt{15}$.

7 Given that $x = 2^{t - 1}$ and $y = 2^{3t}$,

a find expressions in terms of t for

i xy **ii** $2y^2$

b Hence, or otherwise, find the value of t for which

$$2y^2 - xy = 0.$$

8 Solve the equation

$$\sqrt{2}(3x - 1) = 2(2x + 3),$$

giving your answer in the form $a + b\sqrt{2}$, where a and b are integers.

9 Given that $6^{y + 1} = 36^{x - 2}$,

a express y in the form $ax + b$,

b find the value of $4^{x - \frac{1}{2}y}$.

10 Express each of the following in the form $a + b\sqrt{2}$, where a and b are integers.

a $(3 - \sqrt{2})(1 + \sqrt{2})$

b $\dfrac{\sqrt{2}}{\sqrt{2} - 1}$

11 Solve the equation

$$16^{x+1} = 8^{2x+1}.$$

12 Given that

$$(a - 2\sqrt{3})^2 = b - 20\sqrt{3},$$

find the values of the integers a and b.

13 **a** Find the value of t such that

$$(\tfrac{1}{4})^{t-3} = 8.$$

b Solve the equation

$$(\tfrac{1}{3})^{y} = 27^{y+1}.$$

14 Express each of the following in the form $a + b\sqrt{5}$, where a and b are integers.

a $\sqrt{20}(\sqrt{5} - 3)$

b $(1 - \sqrt{5})(3 + 2\sqrt{5})$

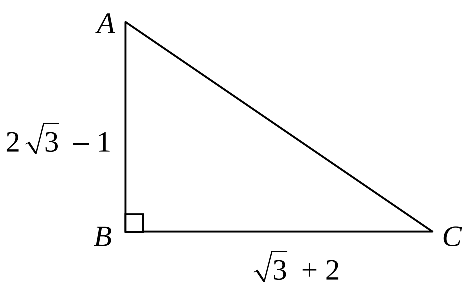
c $\dfrac{1+\sqrt{5}}{\sqrt{5}-2}$

15 Given that $a^{\frac{1}{3}} = b^{\frac{3}{4}}$, and that $a > 0$ and $b > 0$,

a find an expression for $a^{\frac{1}{2}}$ in terms of b,

b find an expression for $b^{\frac{1}{2}}$ in terms of a.

16



In triangle ABC, $AB = 2\sqrt{3} - 1$, $BC = \sqrt{3} + 2$ and $\angle ABC = 90°$.

a Find the exact area of triangle ABC in its simplest form.

b Show that $AC = 2\sqrt{5}$.

c Show that $\tan(\angle ACB) = 5\sqrt{3} - 8$.

17 **a** Given that $y = 2^x$, express each of the following in terms of y.

i 2^{x+2} **ii** 4^x

b Hence, or otherwise, find the value of x for which

$$4^x - 2^{x+2} = 0.$$

18 Given that the point with coordinates $(1 + \sqrt{3}, 5\sqrt{3})$ lies on the curve with the equation

$$y = 2x^2 + px + q,$$

find the values of the rational constants p and q.

ALGEBRA

1 Factorise

a $x^2 + 4x + 3$ **b** $x^2 + 7x + 10$ **c** $y^2 - 3y + 2$ **d** $x^2 - 6x + 9$

e $y^2 - y - 2$ **f** $a^2 + 2a - 8$ **g** $x^2 - 1$ **h** $p^2 + 9p + 14$

i $x^2 - 2x - 15$ **j** $16 - 10m + m^2$ **k** $t^2 + 3t - 18$ **l** $y^2 - 13y + 40$

m $r^2 - 16$ **n** $y^2 - 2y - 63$ **o** $121 + 22a + a^2$ **p** $x^2 + 6x - 72$

q $26 - 15x + x^2$ **r** $s^2 + 23s + 120$ **s** $p^2 + 14p - 51$ **t** $m^2 - m - 90$

2 Factorise

a $2x^2 + 3x + 1$ **b** $2 + 7p + 3p^2$ **c** $2y^2 - 5y + 3$ **d** $2 - m - m^2$

e $3r^2 - 2r - 1$ **f** $5 - 19y - 4y^2$ **g** $4 - 13a + 3a^2$ **h** $5x^2 - 8x - 4$

i $4x^2 + 8x + 3$ **j** $9s^2 - 6s + 1$ **k** $4m^2 - 25$ **l** $2 - y - 6y^2$

m $4u^2 + 17u + 4$ **n** $6p^2 + 5p - 4$ **o** $8x^2 + 19x + 6$ **p** $12r^2 + 8r - 15$

3 Using factorisation, solve each equation.

a $x^2 - 4x + 3 = 0$ **b** $x^2 + 6x + 8 = 0$ **c** $x^2 + 4x - 5 = 0$ **d** $x^2 - 7x = 8$

e $x^2 - 25 = 0$ **f** $x(x - 1) = 42$ **g** $x^2 = 3x$ **h** $27 + 12x + x^2 = 0$

i $60 - 4x - x^2 = 0$ **j** $5x + 14 = x^2$ **k** $2x^2 - 3x + 1 = 0$ **l** $x(x - 1) = 6(x - 2)$

m $3x^2 + 11x = 4$ **n** $x(2x - 3) = 5$ **o** $6 + 23x - 4x^2 = 0$ **p** $6x^2 + 10 = 19x$

q $4x^2 + 4x + 1 = 0$ **r** $3(x^2 + 4) = 13x$ **s** $(2x + 5)^2 = 5 - x$ **t** $3x(2x - 7) = 2(7x + 3)$

4 Factorise fully

a $2y^2 - 10y + 12$ **b** $x^3 + x^2 - 2x$ **c** $p^3 - 4p$ **d** $3m^3 + 21m^2 + 18m$

e $a^4 + 4a^2 + 3$ **f** $t^4 + 3t^2 - 10$ **g** $12 + 20x - 8x^2$ **h** $6r^2 - 9r - 42$

i $6x^3 - 26x^2 + 8x$ **j** $y^4 + 3y^3 - 18y^2$ **k** $m^4 - 1$ **l** $p^5 - 4p^3 + 4p$

5 Sketch each curve showing the coordinates of any points of intersection with the coordinate axes.

a $y = x^2 - 3x + 2$ **b** $y = x^2 + 5x + 6$ **c** $y = x^2 - 9$

d $y = x^2 - 2x$ **e** $y = x^2 - 10x + 25$ **f** $y = 2x^2 - 14x + 20$

g $y = -x^2 + 5x - 4$ **h** $y = 2 + x - x^2$ **i** $y = 2x^2 - 3x + 1$

j $y = 2x^2 + 13x + 6$ **k** $y = 3 - 8x + 4x^2$ **l** $y = 2 + 7x - 4x^2$

m $y = 5x^2 - 17x + 6$ **n** $y = -6x^2 + 7x - 2$ **o** $y = 6x^2 + x - 5$

6 Solve each of the following equations.

a $x - 5 + \frac{4}{x} = 0$ **b** $x - \frac{10}{x} = 3$ **c** $2x^3 - x^2 - 3x = 0$ **d** $x^2(10 - x^2) = 9$

e $\frac{5}{x^2} + \frac{4}{x} - 1 = 0$ **f** $\frac{x-6}{x-4} = x$ **g** $x + 5 = \frac{3}{x+3}$ **h** $x^2 - \frac{4}{x^2} = 3$

i $4x^4 + 7x^2 = 2$ **j** $\frac{2x}{3-x} = \frac{1}{x+2}$ **k** $\frac{2x+1}{x+3} = \frac{2}{x}$ **l** $\frac{7}{x+2} - 3x = 2$

ALGEBRA

1 By completing the square, show that the roots of the equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

2 Use the quadratic formula to solve each equation, giving your answers as simply as possible in terms of surds where appropriate.

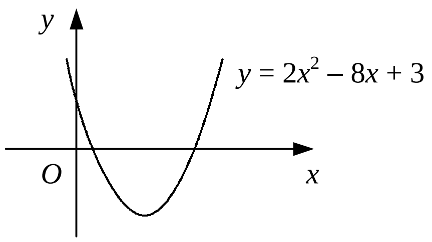
a $x^2 + 4x + 1 = 0$ **b** $4 + 8t - t^2 = 0$ **c** $y^2 - 20y + 91 = 0$ **d** $r^2 + 2r - 7 = 0$

e $6 + 18a + a^2 = 0$ **f** $m(m - 5) = 5$ **g** $x^2 + 11x + 27 = 0$ **h** $2u^2 + 6u + 3 = 0$

i $5 - y - y^2 = 0$ **j** $2x^2 - 3x = 2$ **k** $3p^2 + 7p + 1 = 0$ **l** $t^2 - 14t = 14$

m $0.1r^2 + 1.4r = 0.9$ **n** $6u^2 + 4u = 1$ **o** $\frac{1}{2}y^2 - 3y = \frac{2}{3}$ **p** $4x(x - 3) = 11 - 4x$

3



The diagram shows the curve with equation $y = 2x^2 - 8x + 3$.

Find and simplify the exact coordinates of the points where the curve crosses the x-axis.

4 State the condition for which the roots of the equation $ax^2 + bx + c = 0$ are

a real and distinct **b** real and equal **c** not real

5 Sketch the curve $y = ax^2 + bx + c$ and the x-axis in the cases where

a $a > 0$ and $b^2 - 4ac > 0$ **b** $a < 0$ and $b^2 - 4ac < 0$

c $a > 0$ and $b^2 - 4ac = 0$ **d** $a < 0$ and $b^2 - 4ac > 0$

6 By evaluating the discriminant, determine whether the roots of each equation are real and distinct, real and equal or not real.

a $x^2 + 2x - 7 = 0$ **b** $x^2 + x + 3 = 0$ **c** $x^2 - 4x + 5 = 0$ **d** $x^2 - 6x + 3 = 0$

e $x^2 + 14x + 49 = 0$ **f** $x^2 - 9x + 17 = 0$ **g** $x^2 + 3x = 11$ **h** $2 + 3x + 2x^2 = 0$

i $5x^2 + 8x + 3 = 0$ **j** $3x^2 - 7x + 5 = 0$ **k** $9x^2 - 12x + 4 = 0$ **l** $13x^2 + 19x + 7 = 0$

m $4 - 11x + 8x^2 = 0$ **n** $x^2 + \frac{2}{3}x = \frac{1}{4}$ **o** $x^2 - \frac{3}{4}x + \frac{1}{8} = 0$ **p** $\frac{2}{5}x^2 + \frac{3}{5}x + \frac{1}{3} = 0$

7 Find the value of the constant p such that the equation $x^2 + x + p = 0$ has equal roots.

8 Given that $q \neq 0$, find the value of the constant q such that the equation $x^2 + 2qx - q = 0$ has a repeated root.

9 Given that the x-axis is a tangent to the curve with the equation

$$y = x^2 + rx - 2x + 4,$$

find the two possible values of the constant r.

ALGEBRA

1 Express in the form $(x + a)^2 + b$

a $x^2 + 2x + 4$ **b** $x^2 - 2x + 4$ **c** $x^2 - 4x + 1$ **d** $x^2 + 6x$

e $x^2 + 4x + 8$ **f** $x^2 - 8x - 5$ **g** $x^2 + 12x + 30$ **h** $x^2 - 10x + 25$

i $x^2 + 6x - 9$ **j** $18 - 4x + x^2$ **k** $x^2 + 3x + 3$ **l** $x^2 + x - 1$

m $x^2 - 18x + 100$ **n** $x^2 - x - \frac{1}{2}$ **o** $20 + 9x + x^2$ **p** $x^2 - 7x - 2$

q $5 - 3x + x^2$ **r** $x^2 - 11x + 37$ **s** $x^2 + \frac{2}{3}x + 1$ **t** $x^2 - \frac{1}{2}x - \frac{1}{4}$

2 Express in the form $a(x + b)^2 + c$

a $2x^2 + 4x + 3$ **b** $2x^2 - 8x - 7$ **c** $3 - 6x + 3x^2$ **d** $4x^2 + 24x + 11$

e $-x^2 - 2x - 5$ **f** $1 + 10x - x^2$ **g** $2x^2 + 2x - 1$ **h** $3x^2 - 9x + 5$

i $3x^2 - 24x + 48$ **j** $3x^2 - 15x$ **k** $70 + 40x + 5x^2$ **l** $2x^2 + 5x + 2$

m $4x^2 + 6x - 7$ **n** $-2x^2 + 4x - 1$ **o** $4 - 2x - 3x^2$ **p** $\frac{1}{3}x^2 + \frac{1}{2}x - \frac{1}{4}$

3 Solve each equation by completing the square, giving your answers as simply as possible in terms of surds where appropriate.

a $y^2 - 4y + 2 = 0$ **b** $p^2 + 2p - 2 = 0$ **c** $x^2 - 6x + 4 = 0$ **d** $7 + 10r + r^2 = 0$

e $x^2 - 2x = 11$ **f** $a^2 - 12a - 18 = 0$ **g** $m^2 - 3m + 1 = 0$ **h** $9 - 7t + t^2 = 0$

i $u^2 + 7u = 44$ **j** $2y^2 - 4y + 1 = 0$ **k** $3p^2 + 18p = -23$ **l** $2x^2 + 12x = 9$

m $-m^2 + m + 1 = 0$ **n** $4x^2 + 49 = 28x$ **o** $1 - t - 3t^2 = 0$ **p** $2a^2 - 7a + 4 = 0$

4 By completing the square, find the maximum or minimum value of y and the value of x for which this occurs. State whether your value of y is a maximum or a minimum in each case.

a $y = x^2 - 2x + 7$ **b** $y = x^2 + 2x - 3$ **c** $y = 1 - 6x + x^2$

d $y = x^2 + 10x + 35$ **e** $y = -x^2 + 4x + 4$ **f** $y = x^2 + 3x - 2$

g $y = 2x^2 + 8x + 5$ **h** $y = -3x^2 + 6x$ **i** $y = 7 - 5x - x^2$

j $y = 4x^2 - 12x + 9$ **k** $y = 4x^2 + 20x - 8$ **l** $y = 17 - 2x - 2x^2$

5 Sketch each curve showing the exact coordinates of its turning point and the point where it crosses the y-axis.

a $y = x^2 - 4x + 3$ **b** $y = x^2 + 2x - 24$ **c** $y = x^2 - 2x + 5$

d $y = 30 + 8x + x^2$ **e** $y = x^2 + 2x + 1$ **f** $y = 8 + 2x - x^2$

g $y = -x^2 + 8x - 7$ **h** $y = -x^2 - 4x - 7$ **i** $y = x^2 - 5x + 4$

j $y = x^2 + 3x + 3$ **k** $y = 3 + 8x + 4x^2$ **l** $y = -2x^2 + 8x - 15$

m $y = 1 - x - 2x^2$ **n** $y = 25 - 20x + 4x^2$ **o** $y = 3x^2 - 4x + 2$

6 **a** Express $x^2 - 4\sqrt{2}x + 5$ in the form $a(x + b)^2 + c$.

b Write down an equation of the line of symmetry of the curve $y = x^2 + 4\sqrt{2}x + 5$.

7 $$f(x) \equiv x^2 + 2kx - 3.$$

By completing the square, find the roots of the equation $f(x) = 0$ in terms of the constant k.

ALGEBRA

1 Find the set of values of x for which

a $2x + 1 < 7$ **b** $3x - 1 \geq 20$ **c** $2x - 5 > 3$ **d** $6 + 3x \leq 42$

e $5x + 17 \geq 2$ **f** $\frac{1}{3}x + 7 < 8$ **g** $9x - 4 \geq 50$ **h** $3x + 11 < 7$

i $18 - x > 4$ **j** $10 + 4x \leq 0$ **k** $12 - 3x < 10$ **l** $9 - \frac{1}{2}x \geq 4$

2 Solve each inequality.

a $2y - 3 > y + 4$ **b** $5p + 1 \leq p + 3$ **c** $x - 2 < 3x - 8$

d $a + 11 \geq 15 - a$ **e** $17 - 2u < 2 + u$ **f** $5 - b \geq 14 - 3b$

g $4x + 23 < x + 5$ **h** $12 + 3y \geq 2y - 1$ **i** $16 - 3p \leq 36 + p$

j $5(r - 2) > 30$ **k** $3(1 - 2t) \leq t - 4$ **l** $2(3 + x) \geq 4(6 - x)$

m $7(y + 3) - 2(3y - 1) < 0$ **n** $4(5 - 2x) > 3(7 - 2x)$ **o** $3(4u - 1) - 5(u - 3) < 9$

3 Find the set of values of x for which

a $x^2 - 4x + 3 < 0$ **b** $x^2 - 4 \leq 0$ **c** $15 + 8x + x^2 < 0$ **d** $x^2 + 2x \leq 8$

e $x^2 - 6x + 5 > 0$ **f** $x^2 + 4x > 12$ **g** $x^2 + 10x + 21 \geq 0$ **h** $22 + 9x - x^2 > 0$

i $63 - 2x - x^2 \leq 0$ **j** $x^2 + 11x + 30 > 0$ **k** $30 + 7x - x^2 > 0$ **l** $x^2 + 91 \geq 20x$

4 Solve each inequality.

a $2x^2 - 9x + 4 \leq 0$ **b** $2r^2 - 5r - 3 < 0$ **c** $2 - p - 3p^2 \geq 0$

d $2y^2 + 9y - 5 > 0$ **e** $4m^2 + 13m + 3 < 0$ **f** $9x - 2x^2 \leq 10$

g $a^2 + 6 < 8a - 9$ **h** $x(x + 4) \leq 7 - 2x$ **i** $y(y + 9) > 2(y - 5)$

j $x(2x + 1) > x^2 + 6$ **k** $u(5 - 6u) < 3 - 4u$ **l** $2t + 3 \geq 3t(t - 2)$

m $(y - 2)^2 \leq 2y - 1$ **n** $(p + 2)(p + 3) \geq 20$ **o** $2(13 + 2x) < (6 + x)(1 - x)$

5 Giving your answers in terms of surds, find the set of values of x for which

a $x^2 + 2x - 1 < 0$ **b** $x^2 - 6x + 4 > 0$ **c** $11 - 6x - x^2 > 0$ **d** $x^2 + 4x + 1 \geq 0$

6 Find the value or set of values of k such that

a the equation $x^2 - 6x + k = 0$ has equal roots,

b the equation $x^2 + 2x + k = 0$ has real and distinct roots,

c the equation $x^2 - 3x + k = 0$ has no real roots,

d the equation $x^2 + kx + 4 = 0$ has real roots,

e the equation $kx^2 + x - 1 = 0$ has equal roots,

f the equation $x^2 + kx - 3k = 0$ has no real roots,

g the equation $x^2 + 2x + k - 2 = 0$ has real and distinct roots,

h the equation $2x^2 - kx + k = 0$ has equal roots,

i the equation $x^2 + kx + 2k - 3 = 0$ has no real roots,

j the equation $3x^2 + kx - x + 3 = 0$ has real roots.

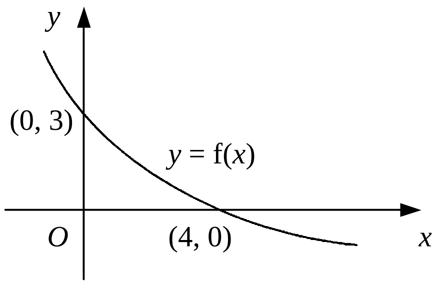
GRAPHS OF FUNCTIONS

1 Describe how the graph of $y = \text{f}(x)$ is transformed to give the graph of

a $y = \text{f}(x - 1)$ **b** $y = \text{f}(x) - 3$ **c** $y = 2\text{f}(x)$ **d** $y = \text{f}(4x)$

e $y = -\text{f}(x)$ **f** $y = \frac{1}{5}\text{f}(x)$ **g** $y = \text{f}(-x)$ **h** $y = \text{f}(\frac{2}{3}x)$

2



The diagram shows the curve with equation $y = \text{f}(x)$ which crosses the coordinate axes at the points (0, 3) and (4, 0).

Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of

a $y = 3\text{f}(x)$ **b** $y = \text{f}(x + 4)$ **c** $y = -\text{f}(x)$ **d** $y = \text{f}(\frac{1}{2}x)$

3 Find and simplify an equation of the graph obtained when

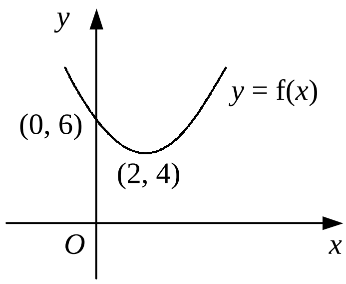
a the graph of $y = 2x + 5$ is translated by 1 unit in the positive y-direction,

b the graph of $y = 1 - 4x$ is stretched by a factor of 3 in the y-direction, about the x-axis,

c the graph of $y = 3x + 1$ is translated by 4 units in the negative x-direction,

d the graph of $y = 4x - 7$ is reflected in the x-axis.

4



The diagram shows the curve with equation $y = \text{f}(x)$ which has a turning point at (2, 4) and crosses the y-axis at the point (0, 6).

Showing the coordinates of the turning point and of any points of intersection with the axes, sketch on separate diagrams the graphs of

a $y = \text{f}(x) - 3$ **b** $y = \text{f}(x + 2)$ **c** $y = \text{f}(2x)$ **d** $y = \frac{1}{2}\text{f}(x)$

5 Describe a single transformation that would map the graph of $y = x^3$ onto the graph of

a $y = 4x^3$ **b** $y = (x - 2)^3$ **c** $y = -x^3$ **d** $y = x^3 + 5$

6 Describe a single transformation that would map the graph of $y = x^2 + 2$ onto the graph of

a $y = 2x^2 + 4$ **b** $y = x^2 - 5$ **c** $y = \frac{1}{9}x^2 + 2$ **d** $y = x^2 + 4x + 6$

7 Find and simplify an equation of the graph obtained when

a the graph of $y = x^2 + 2x$ is translated by 1 unit in the positive x-direction,

b the graph of $y = x^2 - 4x + 5$ is stretched by a factor of $\frac{1}{3}$ in the x-direction, about the y-axis.

c the graph of $y = x^2 + x - 6$ is reflected in the y-axis,

d the graph of $y = 2x^2 - 3x$ is stretched by a factor of 2 in the x-direction, about the y-axis.

8 $\mathrm{f}(x) \equiv x^2 - 4x.$

a Find the coordinates of the turning point of the graph $y = \mathrm{f}(x)$.

b Sketch each pair of graphs on the same set of axes showing the coordinates of the turning point of each graph.

i $y = \mathrm{f}(x)$ and $y = 3 + \mathrm{f}(x)$ **ii** $y = \mathrm{f}(x)$ and $y = \mathrm{f}(x - 2)$ **iii** $y = \mathrm{f}(x)$ and $y = \mathrm{f}(2x)$

9 Sketch each pair of graphs on the same set of axes.

a $y = x^2$ and $y = (x + 3)^2$ **b** $y = x^3$ and $y = x^3 + 4$

c $y = \frac{1}{x}$ and $y = \frac{1}{x - 2}$ **d** $y = \sqrt{x}$ and $y = \sqrt{2x}$

10 a Describe two different transformations, each of which would map the graph of $y = \frac{1}{x}$ onto the graph of $y = \frac{1}{3x}$.

b Describe two different transformations, each of which would map the graph of $y = x^2$ onto the graph of $y = 4x^2$.

11 $\mathrm{f}(x) \equiv (x + 4)(x + 2)(x - 1).$

Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of

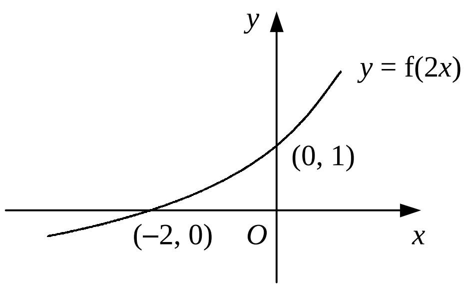
a $y = \mathrm{f}(x)$ **b** $y = \mathrm{f}(x - 4)$ **c** $y = \mathrm{f}(-x)$ **d** $y = \mathrm{f}(2x)$

12 The curve $y = \mathrm{f}(x)$ is a parabola and the coordinates of its turning point are (a, b).

Write down, in terms of a and b, the coordinates of the turning point of the graph

a $y = 3\mathrm{f}(x)$ **b** $y = 4 + \mathrm{f}(x)$ **c** $y = \mathrm{f}(x + 1)$ **d** $y = \mathrm{f}(\frac{1}{3}x)$

13



The diagram shows the curve with equation $y = \mathrm{f}(2x)$ which crosses the coordinate axes at the points $(-2, 0)$ and $(0, 1)$.

Showing the coordinates of any points of intersection with the coordinate axes, sketch on separate diagrams the curves

a $y = 3\mathrm{f}(2x)$ **b** $y = \mathrm{f}(x)$